The Rokhlin Property and Nuclear Dimension

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Introduction

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In fact the Rokhlin property is a strong form of outerness.

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We introduce a new Rokhlin property involving positive elements rather than projections in order to prove some first results.

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T maps T^iE to $T^{i+1}E$ ($i < n - 1$) but it's not clear what T does with $T^{n-1}E$.

However, $\chi_{T^{n-1}E}$ is close to χ_E in L^2.

So T can be approximated by a cyclic shift in L^2.

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- $\|\alpha(p_i) - p_{i+1}\|_2 < \epsilon$, $i = 0, \ldots, n$ **where** $p_n := p_0$
- $p_0 + \ldots + p_{n-1} = 1$

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G a finite group.

Definition (finite groups)

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Definition (single automorphisms)

(1) $\alpha \in \mathrm{Aut}(A)$ has cyclic Rokhlin property if for $\epsilon > 0$, $\mathcal{F} \subset A$, there are infinitely many n and pwo projections $p_0, \ldots, p_{n-1} \in A$ summing to 1 s.t. $\|\alpha(p_i) - p_{i+1}\| < \epsilon \bmod n$, where $p_n := p_0$

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(2) $\alpha \in \mathrm{Aut}(A)$ has Rokhlin property if for $\epsilon > 0$, $n \in \mathbb{N}$, $\mathcal{F} \subset A$ there are $e_0, \ldots, e_{n-1}, f_0, \ldots, f_n \in A$ pwo projections summing to 1 s.t. $\|\alpha(e_i) - e_{i+1}\|, \|\alpha(f_j) - f_{j+1}\| < \epsilon, i = 0, \ldots, n-2, j = 0, \ldots, n-1$ and $\|[e_i, a]\|, \|[f_j, a]\| < \epsilon, \forall a \in \mathcal{F}$

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$$A \longrightarrow F_\lambda \longrightarrow A$$

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(F_λ a finite dim C^*-algebra) as a "noncommutative open cover".
$\mathrm{dr}\,(A)$ and $\mathrm{nucdim}\,(A)$ involve the decomposability of the second map

$$\phi_\lambda : F_\lambda \to A$$

into a sum of orthogonality perserving maps (these are very close to homomorphisms).

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(Except some recent results for minimal homeomorphisms.)

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The same is true for $\mathbb{Z}$-actions with the cyclic Rokhlin property.

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Note that the case $k = 0$ (Rokhlin dimension 0) is equivalent to the cyclic Rohklin property.

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The first point will be illustrated in the next section.

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Clearly, any dynamical system on a compact space which has an irrational rotation as a factor will also have positve Rokhlin property with dimension 1.

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Question: Does α_T satisfy the positive Rokhlin property (of dim. $O(k)$) ?

Seems plausible, but we currently know this only for irrational rotations. (The upper bound k for the Rokhlin dimension of α_T seems too naive.)

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Equip $\mathrm{Aut}(A)$ with the topology of pointwise convergence, more precisely, define basic neighborhoods by

$$V_{\mathcal{F},\epsilon}(\alpha) = \{\beta \in \mathrm{Aut}(A) \mid \|\alpha(a) - \beta(a)\|, \|\alpha^{-1}(a) - \beta^{-1}(a)\| < \epsilon \forall a \in \mathcal{F}\}$$

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If A is separable then $\mathrm{Aut}(A)$ with this topology is a complete metric space.

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Thanks for your attention.